Exercise 77

Find the 50th derivative of $y = \cos 2x$.

Solution

Differentiate the function several times using the chain rule and see if a pattern emerges.

$$y^{(1)} = \frac{d}{dx}(\cos 2x) = -2^1 \sin 2x$$

$$y^{(2)} = \frac{d}{dx}(-2 \sin 2x) = -2^2 \cos 2x$$

$$y^{(3)} = \frac{d}{dx}(-2^2 \cos 2x) = 2^3 \sin 2x$$

$$y^{(4)} = \frac{d}{dx}(2^3 \sin 2x) = 2^4 \cos 2x$$

$$y^{(5)} = \frac{d}{dx}(2^4 \cos 2x) = -2^5 \sin 2x$$

It appears that even derivatives have cosine and that derivatives of order $1 + 4n$ and $2 + 4n$ are negative, where n is an integer. Since $50 = 2 + 4(12)$, the 50th derivative is negative.

$$y^{(50)} = -2^{50} \cos 2x$$